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A nonlinear cointegration approach with applications on structural health monitoring

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1. Introduction

- Structural Health Monitoring (SHM) is an emerging multi-disciplinary technique aiming to assess structural conditions and identify damage.
- One major obstacle to the implementation of SHM is the effect of operational and environmental variabilities, which may significantly alter the dynamic response characteristics of a structure, causing misleading conclusion of the status of the structure.

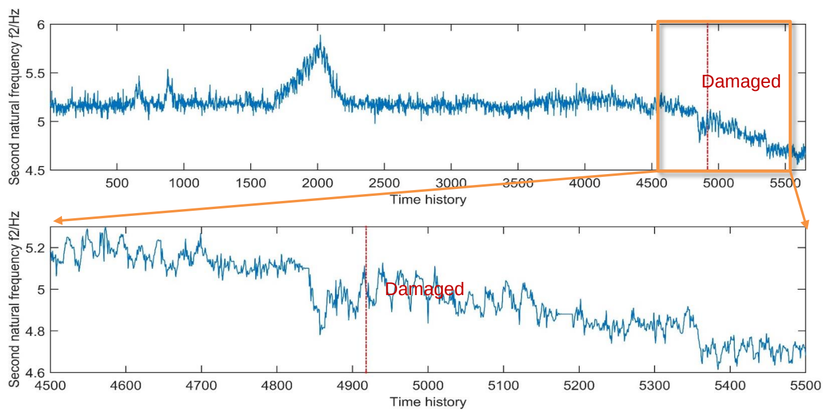


Figure 1. Identified second natural frequency series from Z24 Bridge, even after damage is introduced environmental variation is still dominant, masking damage information

2. Cointegration — an econometric method

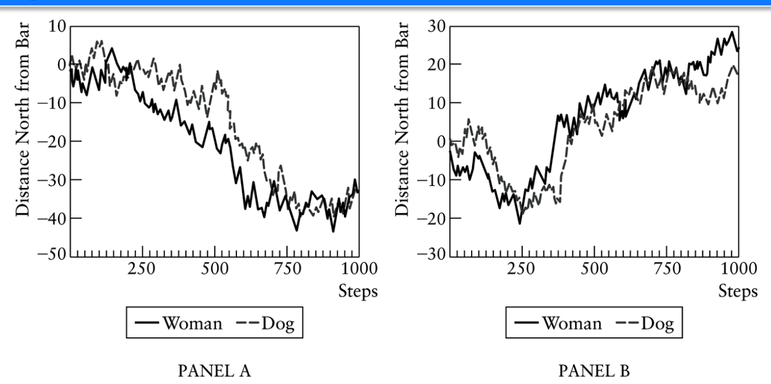


Figure 2. A classical example of "Drunk and Dog" illustrating cointegrating relationships. Panel A and Panel B are simulated paths for the drunk and his dog, horizontal axis tracks their steps from 1 to 1000, vertical axis measures how far they have each wandered from the local pub.

- $Y_t^A, X_t^A, Y_t^B, X_t^B$: the distance series of the drunk and his dog in Panel A and B respectively
- Spurious regression ——$Y_t^A = \beta Y_t^B + \varepsilon_t$, Y_t^A and Y_t^B are not correlated
- Cointegrating regression —— $Y_t^A = \beta X_t^A + \varepsilon_t$, Y_t^A and X_t^A share common trend/long run equilibrium
- Error correction model——$\Delta Y_t = \alpha(Y_{t-1} - \beta X_{t-1}) + \sum_{i=1}^{k} \Delta Y_{t-k} + \varepsilon_t$ model both long run and short run equilibrium

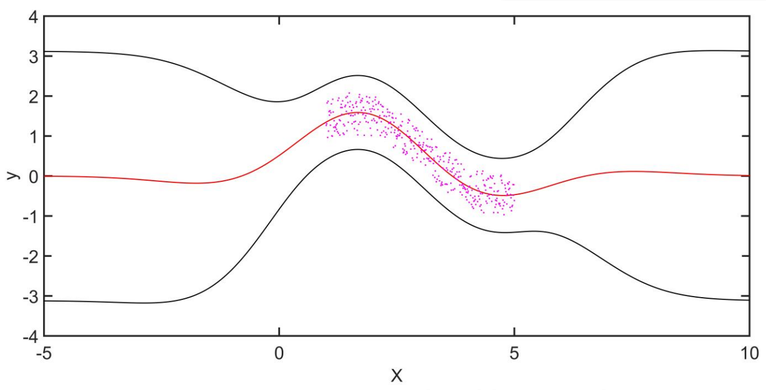
3. Nonlinear cointegration with Gaussian Process

The *linear* nature of coingration, this technique may be restrictive both in the context of economics and engineering, where systems often suggest *nonlinear* responses to unknown parameters.

Linear cointegration: $Y_t = \beta \cdot X_t + \varepsilon_t$

Nonlinear cointegration: $Y_t = f(X_t) + \varepsilon_t$

Proposed method: Gaussian Process as the nonlinear $f(-)$ $Y_t = GP(X_t) + \varepsilon_t$



Consider a nonlinear regression problem, a function $f(-)$ with *confidence intervals* can be learnt from data $D = \{X, y\}$

Figure 3. Gaussian process with confidence intervals

A Gaussian process defines a distribution over functions $p(f)$, and finite number of which are jointly Gaussian distributed. $p(f)$ can be determined in a Bayesian framework:

$$p(f|D) = \frac{p(f)p(D|f)}{p(D)}$$

4. Application case: SHM of the Z-24 Bridge

- The SHM of the Z-24 Bridge in Switzerland is a benchmark study in the field
- one year monitoring study, from 11 November 1997 to 11 September 1998.
- the monitoring campaign introduced artificial damage

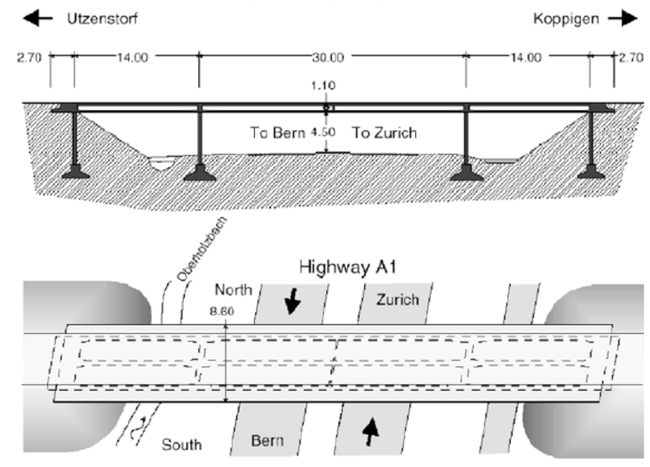


Figure 4. The Z-24 Bridge: longitudinal section and top view

Date	Damage Scenario	Day Index
10/08/98	Settlement of Pier, 20 mm	272
12/08/98	Settlement of Pier, 40 mm	274
17/08/98	Settlement of Pier, 80 mm	279
18/08/98	Settlement of Pier, 95 mm	280
19/08/98	Tilt of Foundation	281
20/08/98	3rd Reference Point	281

Table 1. Progressive damage test scenarios and test dates

- Figure 5 shows the first four natural frequencies with respect to observation number, the data set is separated into two parts: training and test.
- the red dotted line shows the day that the first kind of the damage scenarios was implemented(at data point 4918)

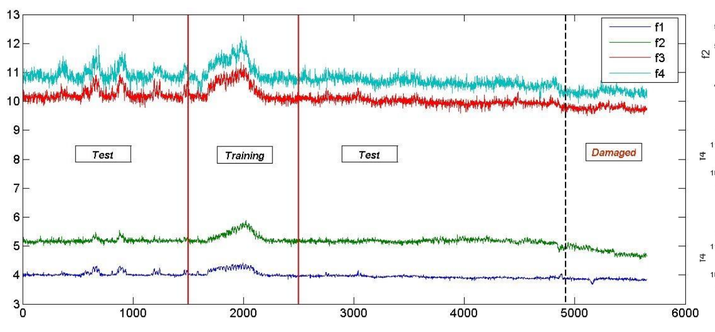


Figure 5. The first four natural frequencies identified from the Z-24 Bridge.

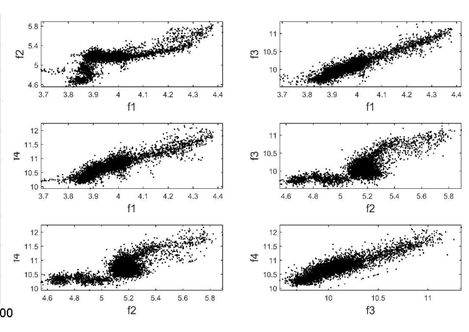


Figure 6. Mutual relationships of the first four natural frequencies.

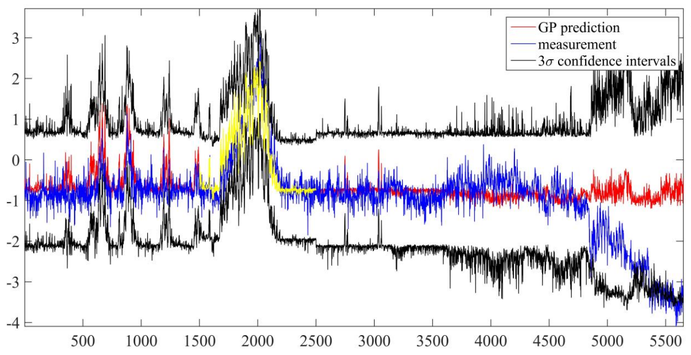


Figure 7. Gaussian process regression Model $f2 = gp(f1, f3, f4)$ prediction performance comparing with real measurements.

- The blue line: original data; the red line: GP prediction; the black line: confidence intervals; the yellow section: training set
- Generally, GP models follow the trend of measurement data very well, the environmentally induced variations are also well predicted.

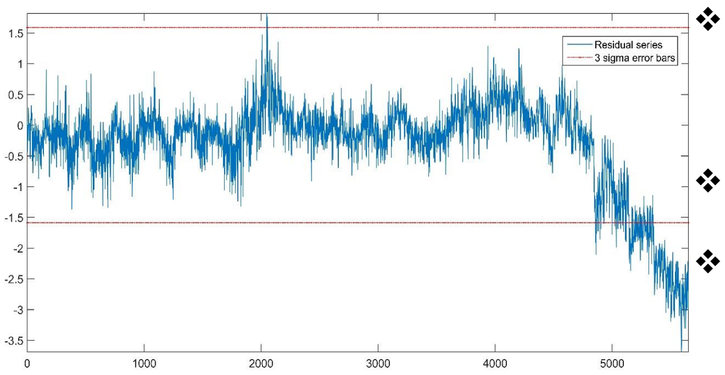


Figure 8. Gaussian process regression Model $f2 = gp(f1, f3, f4)$ residual series

- The upper and lower lines represent the training residual mean plus or minus three times standard deviations
- The residual exceed the boundary after the damage is introduced
- Combining Table 1, the damage point corresponds to the day index 269~270 (7~8 August, 1998), around these dates, the damage scenario 'Settlement of Pier' was implemented.

5. Conclusion

- Cointegration: an effective way to eliminate effect of environmental and operational variation
- Gaussian process regression performs well as a nonlinear cointegrating function
- Nonlinear cointegration method can be utilized in SHM context
- One direct benefit of proposed method is no necessary need for EOV measurements

References

[1] E J Cross K Worden and Q Chen 2011 "Cointegration: a novel approach for the removal of environmental trends in structural health monitoring data" Proc. R. Soc. A Math. Phys. Eng. Sci. vol **467** no 2133 pp 2712–2732

[2] C Rasmussen 2006 *Gaussian processes for machine learning* (MIT Press)